ON THE FORMATION OF PLANETS

AND SATELLITES

NSG-540

J. Anthony Burke

Astrophysics Institute

Brandeis University

May 1967

CONTENTS

ABSTRACT

A model of planetary formation is suggested, whereby gas streaming more or less radially from a protosun interacts with a distant solar nebula in the plane of the ecliptic. A large ring of material is detached from the solar nebula. As this ring moves inward it successively divides: in each event a ring is left behind. The spacing of the series of concentric rings so formed appears to approximate the observed spacing of the planets.

The model when applied to protoplanets surrounded by miniature "solar nebulae" describes a similar process. The rings thus generated are taken to provide the material for satellite formation. The spacing of the satellites of Jupiter, Saturn, and Uranus which show evidence of uniform development is predicted to be about the same as that of the planets. Observations lend credence to this suggestion.

I - INTRODUCTION

Bode's empirical law has never pretended to explain the spacing of planets on the basis of a more fundamental theory, but merely attempts to describe their locations in an ad hoc manner. Subsequent work on cosmogony has generally concentrated on explaining the origin of the planetary system as a whole, and does not customarily predict the sizes of planetary orbits. Thus, the theories of Kant (1755), Laplace (1796), Whipple (1948), Hoyle (1960), and Cameron (1962) consider the problems of formation of a solar nebula, its properties, such as mass distribution, temperature, and composition, its interaction with the magnetic field connecting it to a collapsing protosun, and its condensation into planets. The theories of von Weizsäcker (1944), Berlage (1948), Alfvén (1954), and Schmidt (1959) contain parameters which predict specific spacings for clumps of planetary material, and may therefore be regarded as possible explanations of Bode's Law.

A characteristic feature of most cosmogonies seems to be that of regarding the planets as formed from a medium which is either left in place by a contracting protosun, or produced by material ejected from the protosun. The dominant theme is one of stationary orbits or radial outflow. Angular momentum transfer is from the sun outwards.

We would like to suggest, in this paper, a somewhat different approach. That is, we propose to outline a model in which material flowing out from the protosun (hereafter referred to as the "sun") interacts with the material of a distant solar nebula. In this interaction the solar material absorbs, rather than supplies, angular momentum, thereby causing the nebula to move inwards to the positions of the planetary orbits. As it moves inwards it splits up into concentric rings of material moving in orbit around the sun. In Section III we will estimate the spacing of the rings and thus provide a derivation of Bode's Law. The question of the condensation of these rings into protoplanets we will not consider. However, once a protoplanet has formed, in Section IV we propose a mechanism for the formation of satellites.

The theory which contains elements closest to those of our present model is that of Whipple (1948). In his picture clouds of gas and dust in motion about a protosun accrete material from the surrounding region. The corresponding drag produced causes these clouds to spiral inwards as they become more massive, and makes their orbits circular. They finally end up in the positions of (and become) the present planets. The initial rotation of the protosun is postulated to be small, and thus there are no problems associated with removing angular momentum.

We will conclude with a few brief remarks on other astrophysical situations in which the salient features of this model may be expected to appear.

II - MASS LOSS FROM PROTOSUN

There are many well known or suspected forms of mass loss from stars, ranging from quiet stellar winds to catastrophic supernovae explosions (Deutsch, 1960; Weymann, 1963). We briefly review here three processes which are possibly relevant to a protosun evolving toward the main sequence.

Solar Wind

Mass loss in the form of the solar wind has now become an established phenomenon (Parker, 1963). This more or less isotropic flow carries a flux of about 10^{12} g/sec. at a velocity of hundreds of kilometers per second. The loss of material by the sun during its entire main sequence lifetime will be less than .1% of its mass.

Parker notes that a stellar wind is likely whenever atmospheric turbulence or fluid motion is present at the surface of a star. The energy necessary to produce the solar wind is thought to be provided by the solar hydrogen convection zone. Iben's (1965) models of stars near one solar mass approaching the main sequence possess a surface convective zone: indeed, some are convective throughout. Therefore it is reasonable to expect that a protosun may have a solar wind, of unknown strength.

The angular momentum per unit mass carried by the solar wind is small compared to that of matter in circular orbit at planetary distances. For example, the earth with an orbital velocity of 30 km/sec. has 20 times as much angular momentum per unit mass as the solar wind at 1 A.U. which has an azimuthal velocity of about 1.5 km/sec. (Kraft, 1967). The angular momentum of planets further out increases as the square root of their orbital radii, while that of the solar wind can only remain constant. Any changes in solar wind angular momentum must occur relatively close to the sun where magnetic coupling can be effective.

A solar wind thus provides a weak source of high velocity, low angular momentum material.

Centrifugal Mass Loss

If the sun is formed from a cloud with rotation, then as it contracts upon its approach to the main sequence conservation of angular momentum requires its rotational speed to increase considerably. There is subsequently a good possibility that rotational instability will develop, causing material to be ejected or abandoned in the equatorial plane (Jeans, 1929; Limber, 1967). Limber in his discussion of this mass loss finds the flow in some ways similar to the solar wind. However, near the sun the ejected material will be in nearly circular orbits: centrifugal force will balance gravitational attraction. In that case only a small pressure gradient is required to drive the outward flow. Further from the sun the flow becomes relatively radial. The angular momentum per unit mass of the ejected material will not be much larger than that of material in Keplerian orbit at the solar surface, and this becomes quite small when compared to the angular momentum per unit mass of circular motion at planetary distances. Even if material is ejected already possessing escape velocity its angular momentum is increased by a factor of, at most, only $\sqrt{2}$.

A feature of this mechanism of mass ejection which is particularly relevant to the ensuing discussion is the concentration of the outward mass flow toward the plane of the ecliptic.

T Tauri Stars

There is evidence for rapid loss of mass by T Tauri stars, which are generally considered to represent stars undergoing the pangs of birth. Kuhi (1964) has analyzed line profiles in spectra of eight T Tauri type stars. From his relatively detailed treatment of these spectra we may make the following generalizations:

(a) The spectral classes are late, around G or K. The effective surface temperatures are about 4000-5000° K. The masses are between .6 and $4M_\odot$. The radii are around $3R_\odot$.

(b) There is radial outward flow of material. The velocities at the stellar surface are not greatly different from escape speed, which is around 200-400 km/sec.

(c) The rate of mass loss from this group of stars is quite high, being $1/3\text{-}6\times10^{-7} M_\odot$ per year. The total loss of mass during the T Tauri stage ranges up to $.4M_\odot$ (for a $1M_\odot$ star).

The less specific results of Herbig (1962) confirm the high rate of mass loss in T Tauri stars. He reports fluxes of up to $5 \times 10^{-6} M_{\odot}$ per year.

As far as the angular momentum of the mass flow is concerned, it seems safe to say that it cannot exceed that of centrifugal ejection (maybe this *is* centrifugal ejection), which, as we noted previously, is small.

III - RING FORMATION

Initial Conditions

Whereas the three cases of mass outflow just discussed are all characterized by small angular momenta, this is hardly the only possibility. A feature common to the theories of Hoyle and Alfvén is the principle of magnetic braking, whereby angular momentum is transferred from a rapidly rotating protosun to the solar nebula, or to mass ejected by the protosun. Magnetic fields provide the coupling mechanism. The ejected material remaining in orbit around the sun later condenses to form protoplanets.

Clearly in these models, and in Schatzman's (1962) flare mechanism for loss of spin large amounts of angular momentum are imparted to the outward mass flux. However, in the present model we are going to presume that either the sun is initially slowly rotating, (as Whipple, 1948), or that rotational braking occurs independently of planetary formation, or that braking is mostly completed by the time planetary formation begins (cf. Layzer, 1965).

In addition we require that most of the extra-solar material be located at a large distance from the sun, and in the plane of the ecliptic. The sun should be located in a void, perhaps produced by the collapse of material to make the sun itself. If material is ejected during angular momentum loss it must not remain near the sun but must pass or join or form a solar nebula in orbit beyond the distance of Pluto.

Layzer's (1965) theory of the behavior of magnetoturbulent protostars predicts the emission by the central body of a shell, thickened at its equator, or a ring. The ejected material contains most of the angular momentum of the system. If a solar nebula were formed in this manner, it would not only produce a slowly rotating protosun, but would also leave a void surrounding that sun.

The initial conditions for the model are:

(1) Sun surrounded by void out to a distance of about 50 A. U. Disc shaped solar nebula extending beyond 50 A. U. in circular orbit. Composed of gas and dust.

(2) Fairly large mass flow from the sun. Total flow of around $.1M_{\odot}$ over time scale of about 10 million years. We shall henceforth use the phrase "gas stream" to refer to this mass flow. Gas stream carries angular momentum small compared to that of the solar nebula. Flow velocity slightly in excess of escape, about 300 km/sec. at $3R_{\odot}$. Flow perhaps concentrated toward ecliptic.

Interaction Between Gas Stream and Solar Nebula

A gas stream leaving the solar surface at slightly more than escape velocity (~ 300 km/sec.) will be moving quite slowly (~ 10-15 km/sec.) when it reaches the solar nebula. This velocity is likely to be mildly supersonic, so that a stationary detached shock front will form in front of the solar nebula. Flow behind the front is subsonic, and, in its interaction with the solar nebula, turbulent. This produces mixing with the nebula, thereby augmenting its mass. The self gravitational field of the nebula retains the absorbed part of the gas stream. During the turbulent mixing on the outside of the solar nebula the dust assists in radiating the kinetic-turned-thermal energy of the gas flow.

If the gas stream is subsonic the situation should be similar. Turbulent mixing and gravitational binding will occur. The diminished velocity aids accretion.

As the inner edge of the solar nebula absorbs mass from the protosun its angular momentum remains unchanged, since the gas stream has very little rotation. Therefore the angular momentum per unit mass of the orbiting inner edge of the nebula becomes smaller. This causes it to gradually move inward toward the sun. The amount of solar nebula affected is determined by the scale of the mixing depth. Furthermore, as part of the nebula recedes from the bulk, gravitational forces should aid in pinching off the increasingly tenuous connection between them. If the solar nebula has already fragmented into annular shaped pieces, the detachment is strongly aided. A ring will disconnect from the solar nebula and continue to move inwards. The remaining part of the inside edge of the solar nebula, being shielded from the gas stream by the ring, will remain in place.

Ring Properties

The properties of polytropic gas cylinders and rings have been investigated by Ostriker (1964); non-compressible fluid models have been considered by Randers (1942). These analyses have always assumed a condition of uniform rotation (no shear), and although not exactly applicable to the present model, they may give a general picture of some aspects of ring behavior. If a ring has cross section (2h) small compared to its distance from the sun (r), then the cylindrical polytropes of Ostriker should approximately describe its equilibrium properties. The central temperature for a cylindrical polytrope of molecular hydrogen, of index $n = 3$, is given by

$$T_c \approx 1.2 \times 10^{-15} \lambda$$

If $\lambda \sim 10^{16}$-10^{17} g/cm, then $T_c \sim 12$-$120°$K, the temperature range we would expect for a gas cloud not subjected to ionizing radiation or a strong heating source. The protosun will not violate these conditions, since according to Iben's models it will have a temperature between about 4500 and 8000°K. The range of λ chosen would provide enough mass in a ring at 5 A.U. to manufacture between 2 and 20 Jupiters, the protoplanet mass suggested by Whipple (1964b). If we let $\frac{h}{r} \sim \frac{1}{20}$, then the central density will vary from 5×10^{-9} - 5×10^{-8} g/cm^3 for a ring at $\frac{1}{3}$ A.U. to 5×10^{-12} - 5×10^{-11} g/cm^3 for one at 40 A.U.

Ring Fission

As the ring detached from the solar nebula accretes material and becomes more massive it moves inward. At some point its mass per unit length equals a critical value, $2\lambda_o$, at which time it becomes radially unstable. We now postulate that

(3) Fragmentation of a ring into two concentric, roughly equal parts occurs when its mass per unit length attains a critical value, $2\lambda_o$.

At this point the inner and outer parts of the ring will separate into more or less equal, independent dynamical systems. This means that further material added on the inner edge of the ring will not be mixed into the outer edge. The mass of the inner ring continues to rise, leaving its angular momentum unchanged, and this ring continues to move inward. The outer part of the bifurcated ring, of mass per unit length λ_o, is left behind, shielded from further material addition by the inner ring. This process is then postulated to repeat. The inner ring moves inward until its mass per unit length attains $2\lambda_o$, the critical value for fragmentation. Another ring of mass per unit length λ_o is left behind, and so on.

The question of the conditions under which the fragmentation process (3) actually occurs needs further consideration. Randers (1942) in his discussion of liquid rings investigated their stability. He found them stable to changes in radius (r), including the case where the changes are a function of azimuthal angle, but unstable to azimuthal changes in cross sectional area. If in fact a gaseous polytrope behaves like Randers' liquid model, then, whether stable azimuthal condensations will form depends on the rate of shear (not considered by Randers) compared to the rate of growth of a disturbance. The former is given approximately by

$$\tau_{shear} \sim \sqrt{\frac{r^3}{GM}} \quad ,$$

the latter by

$$\tau_{collapse} \sim \frac{1}{\sqrt{G \bar{\rho}}} \quad ,$$

where M is the solar mass and $\bar{\rho}$ is the average density in the ring. Equating these two time scales yields the relation,

$$\bar{\rho}_{crit} \sim \frac{M}{r^3} \quad .$$

For liquid ring densities larger than $\bar{\rho}_{crit}$ azimuthal instability will occur; for smaller ring densities shearing motions will stretch out incipient azimuthal condensations into radial perturbations. The ratio of critical density to actual density is given by,

$$\frac{\bar{\rho}_{crit}}{\rho_o} \sim \frac{2\pi M}{m} \left(\frac{h}{r} \right)^2 \quad .$$

We see that for $\frac{h}{r} \sim \frac{1}{20}$ and $M >> m$, the tendency to fragment into "sausage links" is not overwhelming. Cook and Franklin (1964) predict the same sort of behavior for incipient azimuthal instabilities in the ring system of Saturn. Such stretched radial divisions will then either disappear immediately, resulting in a single ring again, or be maintained long enough to permit independent behavior, for a time, of the vestigal inner and outer parts of the ring. The longer the time before coalescence, the longer the time for the inner part of the ring to move inward under the influence of the gas stream, thus broadening or finally splitting the ring. Of course, this whole discussion of fission is obviously quite speculative in its application of the liquid, non-shear, model. Gaseous rings may behave in an entirely different manner. Further work on this problem is needed to fully justify assumption (3).

Ring Spacing

We have mentioned that upon fission of a ring into two nearly equal rings the inner one will shield the outer one from further acquisition of matter from the gas flow. Furthermore, the outer ring will be shielded from radiative heating by the protosun, allowing its temperature to drop sharply, and favoring subsequent development into a proto-planet. Whether this shielding will continue to be effective as the inner ring moves further toward the sun must be investigated. The shielding from solar radiation should be nearly complete (for $\lambda \sim 10^{16} - 10^{17}$ g/cm the rings will be opaque [Gaustad, 1963]) since, as seen from the sun, the inner ring will subtend at least as large an angle as the outer. Condensation of the outer ring and increasing heating of the inner one will act to make the shielding more efficient.

Whether there is effective shielding from the influence of the gas stream after the rings have separated a good deal depends roughly on the ratio of transverse to radial gas particle velocity. The transverse velocity, v_t, we may take as given by

$$m_H v_t^2 \sim kT_s$$

where T_s is the temperature of the gas stream. Near the earth's position, where the radial velocity is of the order of 50 km/sec., if T_s is less than about 10^4 °K, which is likely to be the case, the shielding should be effective. Further out, say at the position of Neptune, when the radial velocity will be of order 10 km/sec., for particle shielding to occur T_s must be less than about 300° K, and this may very likely not be true. Therefore, not immediately after separation, but later on when two Jovian planet rings have become separated by quite a distance, the outer ring may be subject to a bit more accretion from the gas stream, causing it to move inward a little more. This additional vulnerability to accretion will be diminished according to the amount that cooling and condensation has lessened the cross section of the outer ring. Cameron (1962) points out that the drop in temperature of a shielded gas and dust mixture will be very rapid, perhaps to 10° K in a million years.

In initial condition (2) we have only stipulated that the angular momentum of the gas stream be small, not necessarily nonexistent. Since in fact some centrifugal ejection or magnetic coupling is likely to occur, the gas stream will carry non-zero rotation. This angular momentum of the flow will be reflected in slight modifications of the ring spacings.

The effect on rings at a large distance will be negligible since there the angular momentum per unit mass contained in the rings is huge. But for rings at the distances of the closer planets the angular momentum contained in accreted matter begins to be more significant in comparison with that of the ring. Since these two momenta are presumably in the same direction the effect will be to cause less inward motion and consequently to diminish spacing between rings near the sun.

Another factor influencing ring spacing will be the change in mass of the protosun resulting from the gas stream. This effect will cause the outer, and older, rings to move outward, their distances increasing in inverse proportion to the change in solar mass. The inner rings will not be so much affected since they will be born at a later epoch when most of the mass loss has already occurred.

Let us compute a numerical model. In addition to initial conditions (1) and (2) and postulate (3) already stated let us assume for simplicity that

(4) Shielding is perfect.

(5) The mass loss from the central body is small enough relative to its total mass so that orbits are not significantly altered (a fair assumption, as we shall see, for the outer couple of rings; a very good assumption for the rest).

The angular momentum, H, of a ring of mass m_o at distance r_o from the sun is given by

$$H^2 \propto r_o m_o^2 .$$

Since as mass is added to a ring, H remains unchanged, we may write

$$\frac{r}{r_o} = \frac{m_o^2}{m^2} \tag{1}$$

The mass per unit length λ is then given by

$$\frac{\lambda}{\lambda_o} = \frac{m}{m_o} \frac{r_o}{r} \tag{2}$$

We consider a ring as having just been formed by division of a larger ring. Then its mass per unit length is λ_o, by assumption (3). What is its new radius, r, when it has achieved a mass per unit length necessary for division again? Combining equations (1) and (2) we obtain

$$\left(\frac{r}{r_o}\right)^3 = \left(\frac{\lambda_o}{\lambda}\right)^2$$

When $\lambda = 2\lambda_o$, then $\frac{r_o}{r} = 1.59 \approx 1.6$

Total Mass Absorbed

On the basis of the previous calculation, and postulate (3) we may compute the total mass, ΔM, absorbed from the gas stream in the formation of 10 proto-planets from a solar nebula. The mass absorbed between the time a ring is newly born, with mass $m_o = 2\pi r_o \lambda_o$, until it in turn divides, having mass $2m_1 = 4\pi r_1 \lambda_o$, is

$$\Delta m = 2m_1 - m_o = m_o \left(2 \frac{r_1}{r_o} - 1 \right).$$

Thus

$$\Delta M = \sum_{n=1}^{10} \left(\frac{r_n}{r_{n-1}} \right)^{n-1} m_o \left(2 \frac{r_n}{r_{n-1}} - 1 \right).$$

For the present case of $\frac{r_n}{r_{n-1}} = \frac{1}{1.6}$

$$\Delta M \doteq 0.7 \, m_o$$

If ring zero is the solar nebula beyond Pluto, at a distance of about 60 A.U. from the sun, and if λ_o has the value of 2×10^{16} g/cm, then

$$\Delta M \approx \frac{M_o}{25} .$$

Let us take the fraction of the flow intercepted and absorbed by rings to be about 1/10. Therefore the total loss of mass by the protosun amounts to about $\frac{2}{5} M_o$. This is quite reasonable, in view of the fact that in the T Tauri stage alone nearly $.4M_o$ may be lost. Any matter lost in earlier stages is likely to be much more concentrated toward the ecliptic and not requiring much of a total flow. It should be furthermore remembered that the present example has put enough mass into the 5 A.U. ring to make 5 Jupiters, enough into the 1 A.U. ring to make 400 Earths, etc.

60% of the total mass absorbed in rings is used in the formation of the first two (Pluto and Neptune), 75% in the formation of the first three. Orbital changes as a result of the changing solar mass will then primarily affect the zero ring (the solar nebula) and the first and second rings. The magnitude of the increase of these ring distances will be only about 10-20%.

Planetary Spacing

In Table I we have listed the planets, including the asteroid belt, and the semi-major axes of their orbits. In the third column the ratios of the axes are tabulated. Within the accuracy of the calculation, the grouping of these ratios between 1.31 and 2.01 (with an average of 1.68) can be considered as moderately good agreement with this model.

Being wary of overinterpreting the data, it might be suggested that the smaller values for the planets close to the sun reflect the effect of the non-zero angular momentum of the gas stream. The smaller ratios for the outer planets might represent the effect of poor shielding dominating over the effect of mass loss of the central body. Pluto's orbit, with its high eccentricity, actually intersecting the orbit of Neptune, makes it difficult to decide whether it should be assigned its own ring. Pluto has probably been severely perturbed, or captured from Neptune (Lyttleton, 1936), and thus its original orbit is unknown.

IV - SATELLITE FORMATION

Initial Conditions

If the above mechanism (or, in fact, any mechanism) causes rings to be formed which condense into proto-planets, then a very similar process is likely to occur at each proto-planet. We adopt as initial conditions a proto-planet surrounded by a void (made empty by collapse of material into the planet) beyond which is a miniature "solar nebula" consisting of some of the unused material left over from planet formation. This material will be in the form of a thin disc, and in circular orbital motion around the planet.

Matter Sources

There are two processes which may serve to provide a source of mass with low angular momentum to this disc.

(a) Matter spun off in the direction of the protoplanet's equator during its contraction.

(b) The gas stream (possibly somewhat diminished by now) from the sun, focussed by the proto-planet's gravitational field, and convergent within and on the disc.

Let us consider a few aspects of each of these mechanisms.

(a) The total mass of the satellites of any planet does not exceed 1/80 of the mass of the planet, and if we exclude Earth, does not exceed 1/750 planet mass. This is a small amount of mass and could easily be removed from the parent planet during its formation. The angular momentum of this material will be small compared with that of the disc around the planet, as in the case of the solar nebula. For closer satellites this may no longer be true, and would cause closer spacing of satellites.

(b) In order to estimate the amount of material from the solar gas stream which might be focussed by a planet assume that all material within a distance, the accretion radius (Hoyle and Lyttleton, 1939), is swept up. (Arguments on whether accretion actually occurs are largely irrelevant to this discussion, since only focussing is needed. Some of the focussed stream will undoubtedly be accreted by the planet.) Let us consider for example Saturn; take its mass to be the current value (the least favorable case), and let the solar gas stream have velocity about 10 km/sec. The accretion radius, a , is given by

$$a = \frac{2Gm}{v^2} ,$$

which for Saturn is 5×10^{-3} A.U. If the solar gas stream is removing mass isotropically from the sun at the rate of, say, $3 \times 10^{-7} M_\odot$ per year, then in about a million years Saturn would focus enough material to form all of its satellites from a miniature "solar nebula."

Shielding of outer rings by inner ones would undoubtedly occur, although perhaps not with great efficiency. The angular momentum of the accreted matter would be nearly zero in the non-rotating frame moving with the planet; the effect of the planets orbital motion would be simply to change the apparent radiant of the gas stream.

Satellite Spacing

The spacing of satellites formed in the above manner would resemble very closely that of the planets. Namely, the same idealized model as used in planetary ring formation predicts a ratio of orbital sizes of successive moons of about 1.6. In Table 2 we have listed the satellites of Uranus, Jupiter, and Saturn, the only planets having more than two. The ring system of Saturn (which would actually represent a primordial ring, much as most planets must have had) is treated as a single satellite, Cassini's division being caused by resonance with Mimas. Several orbital parameters are listed in the table. In column three we list the ratios of semi-major axes of neighboring satellites. Note that for Saturn and Jupiter the satellites fall into two categories: (i) those with direct orbits, small eccentricities, and small inclinations, (ii) those with orbits of high inclination, often retrograde (R), and often of high eccentricity. The former category suggests a more untroubled history, common origin, and common evolution. And it is this same category whose orbital sizes fit the pattern of being in ratios not too different from 1.6. We might say, at this point, that if we were about to embark on a search for new satellites of Saturn we would look between satellites #5 and #6, and perhaps between satellites #7 and #8. On Jupiter we would look between #5 and #1. Tidal forces may somewhat modify the initial spacing of satellites by moving the closer ones outward.

V - FURTHER CONSIDERATIONS

Comet Belt

A comet belt beyond Neptune or Pluto as envisioned by Whipple (1964a) might be the remnants of an additional ring beyond Pluto, or part of the same ring from which Pluto was condensed (if it was not formed as a satellite of Neptune). Or they might be made from ring material left over after planet formation.

Similar Mechanisms

An occurrence of exchange of angular momentum similar to that envisioned in the interaction of a gas stream with the solar nebula has been discussed by Oort (1962). He is concerned with the inward motion of a gaseous spiral arm caused by its absorption of gas of lower angular momentum from an interior arm.

The Poynting-Robertson effect likewise operates with a rotationless radial stream of photons causing an orbiting particle to move inwards.

Gas Flow Around Galactic Center

A situation very similar to that described appears to exist in the region around the center of our galaxy. The common features are,

(a) Near the galactic nucleus gas flows radially outward with velocities around 50-150 km/sec. (Rougoor and Oort, 1960).

(b) Two rings of gas are distinguished. One at around 4 kpc is ionized hydrogen (Westerhout, 1958). The density of gas drops on the central side of this ring. The other ring's inner and outer edges are at 500 and 590 pc. respectively (Rougoor and Oort, 1960). Within this ring the density drops sharply until at a distance of about 300-350 pc. the outer edge of a rotating disc appears. One, or both, of these rings may have been formed by the radial flow in the same manner in which planetary rings may be formed by a solar gas stream.

This author is now further investigating detailed models of flows such as occur near the galactic center. It is perhaps not unreasonable to speculate that other rings, such as around, for example, the galaxy NGC 4612 are produced by similar interactions between ejected gas and a surrounding cloud.

VI - SUMMARY

We have proposed a model to describe the formation and spacing of proto-planetary rings, and proto-satellite rings. The three major assumptions of the model were,

(1) Protosun surrounded by void out to 50 A. U. Solar nebula beyond that.

(2) Radial flow of material from protosun carries very little angular momentum.

(3) Rings formed as a result of interaction of gas flow with solar nebula will divide into two roughly equal, concentric parts.

The absorption of gas by the solar nebula from the radial flow causes part of the nebula to move inward. As it progresses toward the sun it repeatedly divides according to assumption (3), leaving a series of concentric rings in orbit. The ratios of the distances from the sun of successive rings is calculated to be about 1.6. This figure agrees well with the observed planetary spacings, thus providing a more fundamental derivation of a modified Bode's law.

A similar analysis was applied to the problem of satellite formation, yielding the same spacing factor of 1.6. There is good agreement with the observed positions of the satellites of Jupiter, Saturn and

Uranus if we consider only those satellites whose orbits are direct and coplanar.

I am pleased to acknowledge helpful discussions with R. E. Berendzen, A. J. Kalnajs, and R. P. Stefanik. This work was supported by National Aeronautics and Space Administration Grant NsG 540.

TABLE 1. Planets

Planet	Orbit Semi-Major Axis	Ratios
Mercury	.387 A.U.	1.87
Venus	.723	1.38
Earth	1.000	1.52
Mars	1.524	1.84
Asteroids	2.8	1.85
Jupiter	5.203	1.84
Saturn	9.539	2.01
Uranus	19.191	1.56
Neptune	30.071	1.31
Pluto	39.518	

TABLE 2. Satellites (Allen 1963)

Satellite	Orbit Semi-Major Axis	Ratios	Orbit Eccentricity	Orbit Inclination
Jupiter				
5	181x10^3km.	2.33 (=1.53^2)	.003	.4°
1 Io	422	1.59	small	0
2 Europa	671	1.60	small	0
3 Ganymede	1070	1.76	small	0
4 Callisto	1883		small	0
6	11470		.158	28
7	11740		.206	26
10	11850		.135	28.5
12	21200		.16	33 (R)
11	22560		.207	16.5 (R)
8	23500		.40	33 (R)
9	23700		.27	25 (R)
Saturn				
Rings	72-137(105Av)			
1 Mimas	186	1.77	.020	1.5
2 Enceladus	238	1.28	.004	0.0
3 Tethys	295	1.24	.0	1.1
4 Dione	377	1.28	.002	0.0
5 Rhea	527	1.40	.001	.3
6 Titan	1222	2.32 (=1.52^2)	.029	.3
7 Hyperion	1481	1.21	.104	.5
8 Iapetus	3560	2.40 (=1.55^2)	.028	14.7
9 Phoebe	12950		.163	30 (R)
Uranus				
5 Miranda	128	1.50	< .01	
1 Ariel	192	1.39	.003	0
2 Umbriel	267	1.64	.004	0
3 Titania	438	1.34	.002	0
4 Oberon	586		.0007	0

REFERENCES

Alfvén, H. (1954). On the Origin of the Solar System. Oxford.

Allen, C. W. (1963). Astrophysical Quantities. Athlone Press, London.

Berlage, H. P., Jr. (1948). Proc. Koninkl. Ned. Akad. Wetenschap. (Amsterdam) 51, 965.

Cameron, A. G. W. (1962). Icarus I, 13.

Cook, A. F. and Franklin, F. A. (1964). A. J. 69, 173.

Deutsch, A. (1960). I.A.U. Symposium No. 12, Aerodynamic Phenomena in Stellar Atmospheres.

Gaustad, J. E. (1963). Ap. J. 138, 1050.

Herbig, G. H. (1962). Advances in Astronomy and Astrophysics 1, 89.

Hoyle, F. and Lyttleton, R. A. (1939). Proc. Camb. Phil. Soc. 35, 405.

Hoyle, F. (1960). Quart. Journ. R.A.S. 1, 28.

Iben, I., Jr. (1965). Ap. J. 141, 993.

Jeans, J. (1929). Astronomy and Cosmogony. Dover, New York, 1961.

Kant, I. (1755). Allgemeine Naturgeschichte und Theorie des Himmels.

Kraft, R. (1967). Colloquium, Harvard College Observatory.

Kuhi, L. V. (1964). Ap. J. 140, 1409. Also Ap. J. 143, 991, 1966.

Laplace, P. S. de (1796). Exposition du Systême du Monde. Paris.

Layzer, D. (1965). Ap. J. 141, 837.

Limber, D. N. (1967). Ap. J. 148, 141.

Lyttleton, R. A. (1939). M.N.R.A.S. 97, 108.

Oort, J. H. (1962). In The Distribution and Motion of Interstellar Matter in Galaxies. Ed. L. Woltjer. New York, W. A. Benjamin. p-241.

Ostriker, J. (1964). Ap. J. 140, 1056 and 1067.

Parker, E. N. (1963). Interplanetary Dynamical Processes. Interscience, New York.

Randers, G. (1942). Ap. J. 95, 88.

Rougoor, G. W. and Oort, J. H. (1960). Proc. Nat. Acad. Sci. 46, 1.

Schatzman, E. (1962). Ann. d'Astro. 25, 18.

Schmidt, O. (1959). A Theory of the Origin of the Earth; Four Lectures. Lawrence and Wishart, London.

von Weizsäcker, C. F. (1944). Zeit. Für Astrophys. 22, 319.

Westerhout, G. (1958). B.A.N. 14, 215 and 261.

Weymann, R. (1963). Annual Review of Astronomy and Astrophysics 1, 97.

Whipple, F. L. (1948). Harvard Observatory Monographs No. 7.

Whipple, F. L. (1964a). Proc. Nat. Acad. Sciences 51, 711.

Whipple, F. L. (1964b). Proc. Nat. Acad. Sciences 52, 565.